Chapter Two

Discrete time signal

1-1 Representation of Discrete Time Sequences

The discrete time sequence is denoted by x (n) as shown in Figure (1). Here 'n' is the corresponding number of the sample, in the given diagram the value of n varies from -3 to + 3. On the Y-axis, the amplitude of signal is plotted. The signal is having some amplitude at each value of n. There are three representations of discrete signals:

- Functional representation
- Tabular representation
- Sequence representation

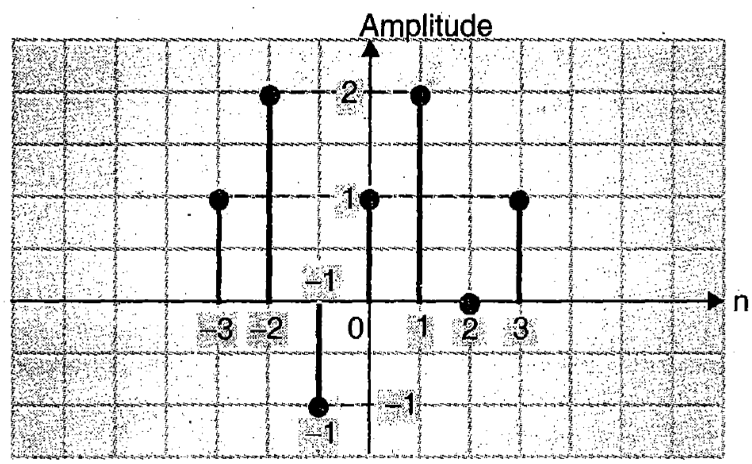


Figure 1: A discrete signal.

$$x(n) = \begin{cases} 2 & \text{for } n = -2, 1 \\ 1 & \text{for } n = -3, 0, 3 \\ 0 & \text{for } n = 2 \\ -1 & \text{for } n = -1 \end{cases}$$

$$x(n) = \{1, 2, -1, \underset{\uparrow}{1}, 2, 0, 1\}$$

n	....	-3	-2	-1	0	1	2	3
x(n)	....	1	2	-1	1	2	0	1

Figure 2 : Types of representations of discrete signals. Functional, sequence and tabular.

1-2 Basic Sequence Types

The most commonly used sequences are:

- Unit Impulse or Delta
- Unit Step
- Unit Ramp
- Exponential
- Sinusoidal
- Complex Exponential

1-2.1 Unit Impulse or Delta Function

A Unit impulse function is denoted by δ (n), its amplitude is 1 at n=0 and it is zero at all other instances. It is represented by:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

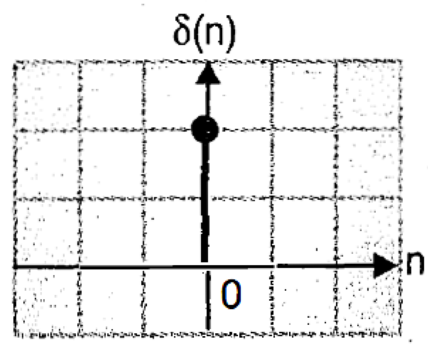


Figure 3: Unit Impulse

1-2.2 Unit Step

A Unit function is denoted by U (n) and its value is unity (one) for all values of n, and is zero for all negative values of n. it is written as $u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$:

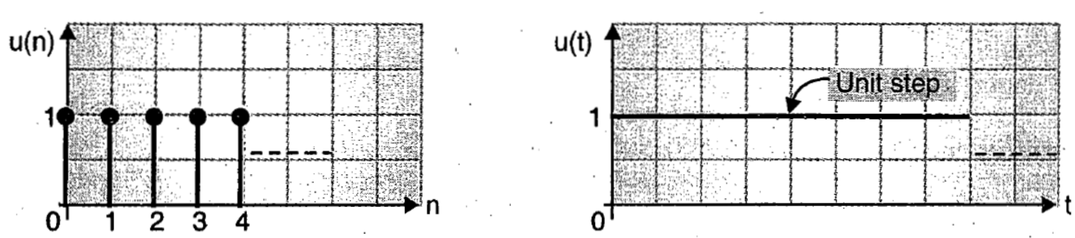


Figure 4: Unit step sequence and function.

1-2.3 Unit Ramp

A Unit function is denoted by U_r (n) and its value increases linearly with the value of n and is zero for all negative n values.

$$u_r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

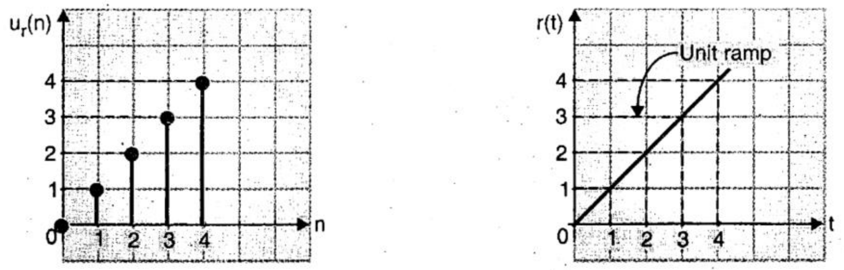


Figure 5 : Unit Ramp.

1-2.4 Exponential Signal

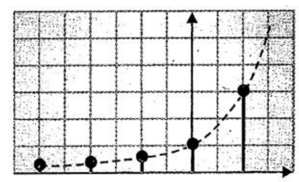
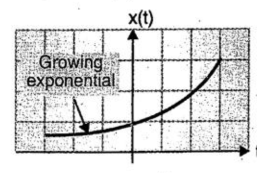
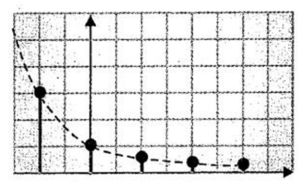
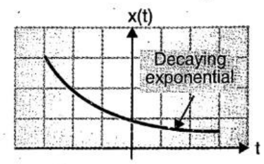
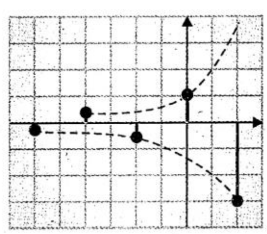
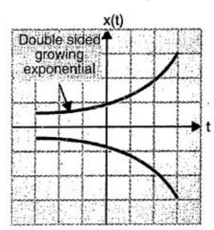
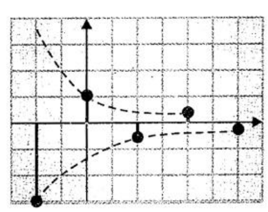
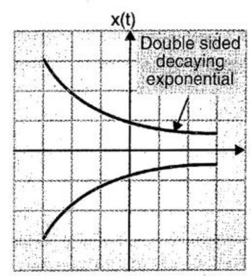
Sr. No.	Type	D. T. Waveform	C. T. Waveform
5.	Exponential		
(i)	When a > 1	 $x(n) = a^n$	 $x(t) = a^t$
(ii)	When 0 < a < 1	 $x(n) = a^n$	 $x(t) = a^t$
(iii)	When a < −1	 $x(n) = a^n$	 $x(t) = a^t$
(iv)	When −1 < a < 0	 $x(n) = a^n$	 $x(t) = a^t$

Figure 6 : Cases of Exponential signals x (n) = a^n

1-2.5 Sinusoidal Signal

$x(n) = A \sin \omega n$ Here A = Amplitude ω = Angular Frequency $= 2\pi f$

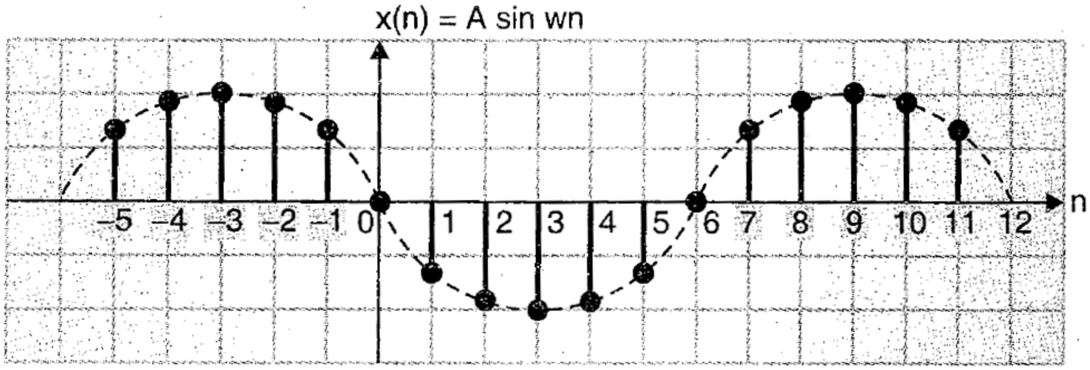


Figure 7: DT Sinusoidal waveform.

1-2.6 Complex Exponentials

The complex exponential function will become a critical part of your study of signals and systems. Its general discrete form is written as: Ae^{sn} Where, $s=\sigma+i\omega$. Here we can use Euler's formula which is:

$e^{i\omega n} = e^{i(\omega+2\pi)n}$ And $e^{jx} = \cos(x) + j\sin(x)$

There will be two cases for the complex exponentials according to the value of e:

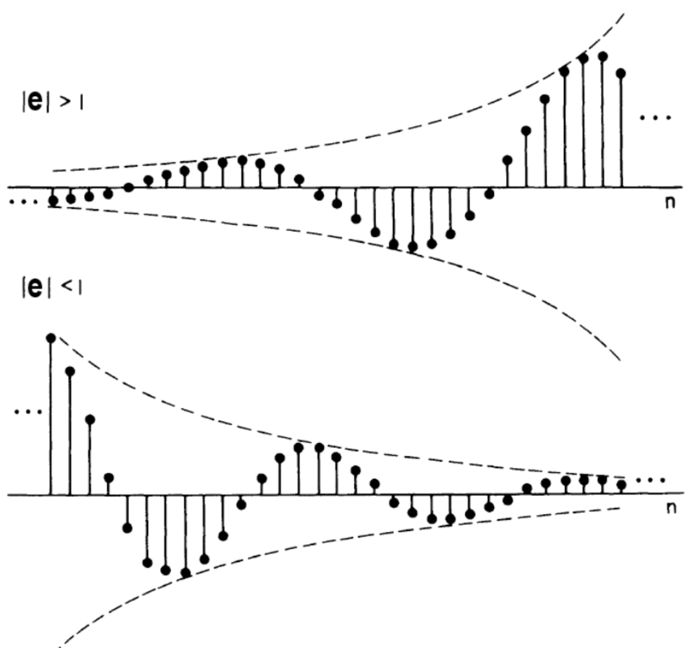


Figure 8: DT Complex exponential.

1-3 Basic Operations on Sequence

Many times it is necessary to modify the original signal. This modification is achieved by performing different operations on given discrete-time signal. Some of these operations are:

- Time Delay
- Time Advance
- Time Folding
- Time Scaling (Up and Down Sampling)
- Amplitude Scaling (Amplification and Attenuation)
- Signals Addition
- Signals Multiplication

1-3.1 Time Delay

$$\therefore \quad x(n) \rightarrow \text{Original sequence}$$

$$\text{and} \quad x(n-k) \rightarrow \text{Original sequence delayed by k samples.}$$

Example: Let the given signal by: $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ which is shown in Figure (9)

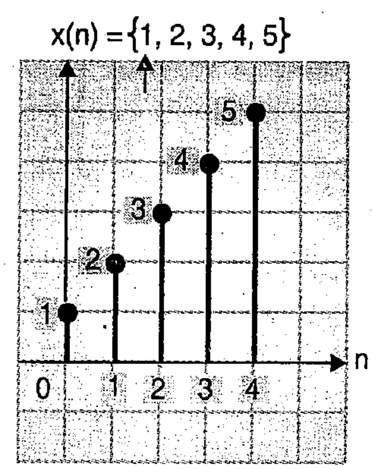


Figure 9: A Discrete Signal

We can write the delayed sequence as:

$$x(n-k) = x(n-2) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5\}$$

The Delayed version is shown in Figure (10).

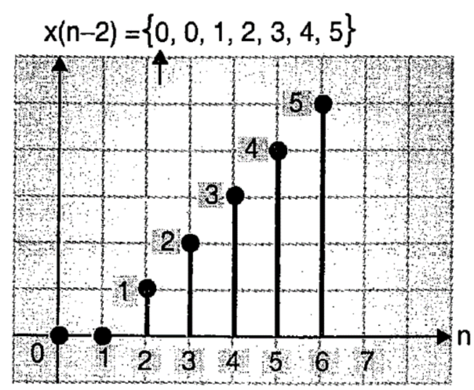


Figure 10: Delayed by 2.

1-3.2 Time Advance

We can write advanced sequence as:

$$x(n+k) = x(n+2) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$$

Which is shown in Figure (11).

1-3.3* Time Folding*: It is the Reflection as: $x(-n) = \{5, 4, 3, 2, \underset{\uparrow}{1}\}$

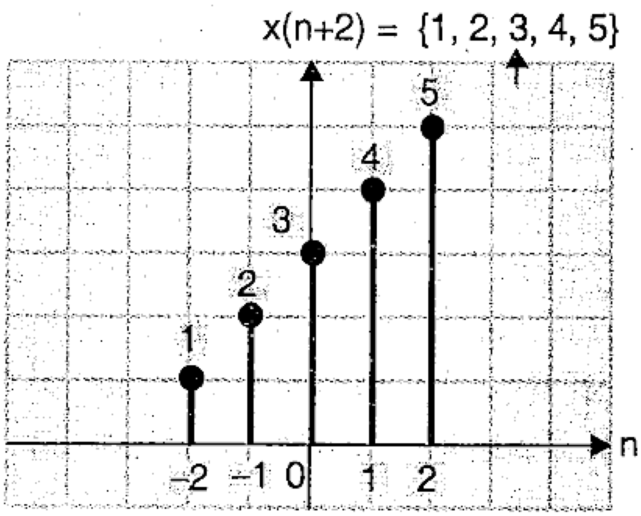


Figure 11: Advanced Signal.

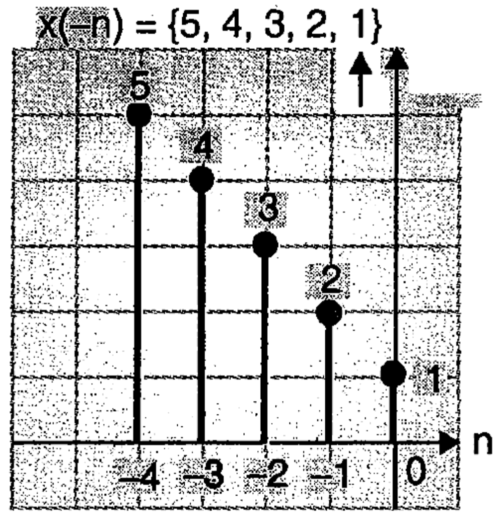


Figure 12: Folded Signal

The folded signal can be delayed and advanced as shown in Figure (13) and Figure (14) respectively.

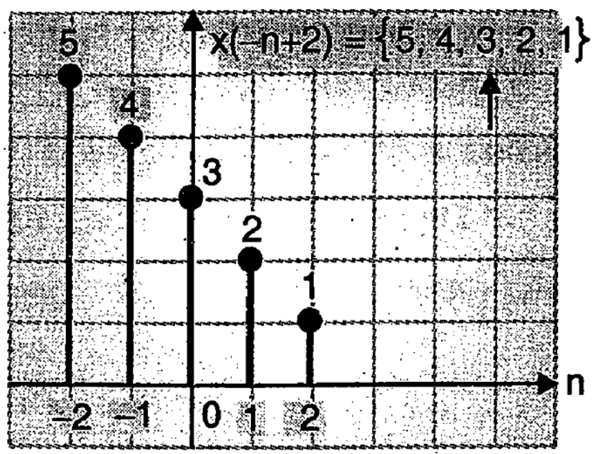


Figure 13: Delayed Folded Signal.

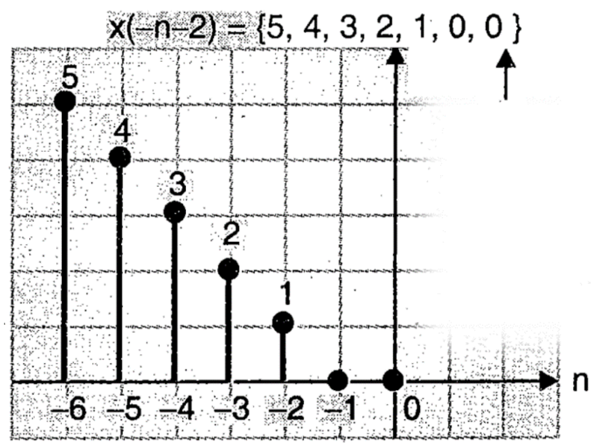


Figure 14: Advanced Folded Signal.

1-3.4 Time Scaling

There are two types of time scaling, down and up scaling. Scaling call sometimes sampling. **Down sampling** as called Compression, scaling down by 2 is written as: that mean $y(n) = x(2n)$ every two samples one goes out. For the signal of Figure (9) the output will become: $y(n) = x(2n) = \{\underset{\uparrow}{1}, 3, 5, 0, ...\}$ Which is shown in Figure (15).

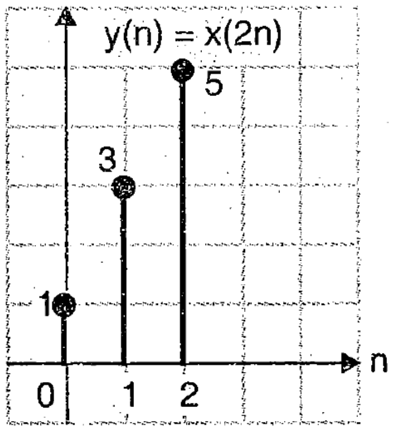


Figure 15: Down sampling

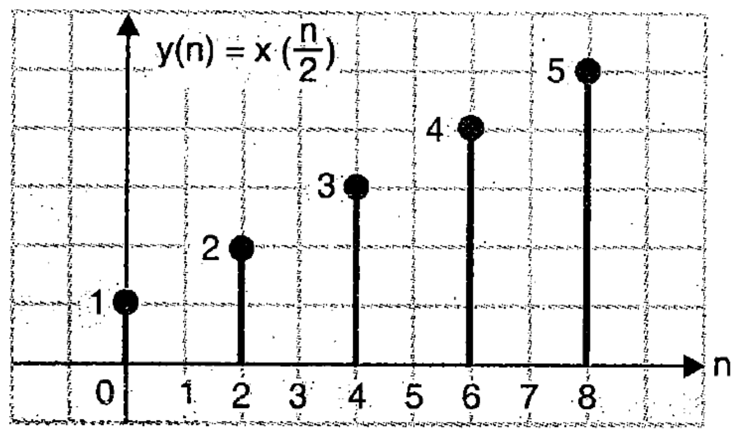


Figure 16: Up Sampling

Up sampling as called Expansion, scaling up by 2 is written as: $y(n) = x\left(\frac{n}{2}\right)$

There will be zeros between each sample as shown in figure (16)

1-3.5 Amplitude Scaling

An **Amplification** operation is, a sample multiplication, denoted for example by y (n) = 2 * x (n), for the same x (n) of Figure (9) the output will be as Figure (17).

While an **Attenuation** operation is dividing by a number, and is denoted by:

Let $y(n) = \frac{x(n)}{2}$

For the same signal of Figure (9), the output will be as in figure (18).

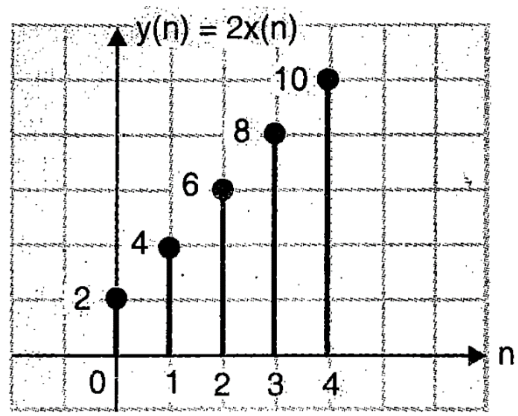


Figure 17: Amplified by 2.

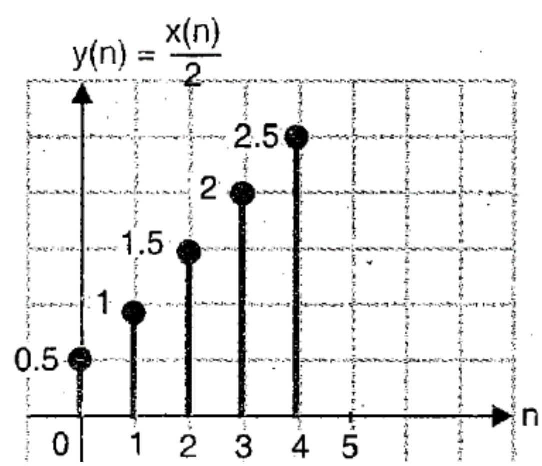


Figure 18: Attenuated by 2.

1-3.6 Signals Addition

Consider the two sequences: $x_1(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$ and $x_2(n) = \{2, 2, \underset{\uparrow}{0}, 2, 2\}$

$$\text{Let} \quad y(n) = x_1(n) + x_2(n)$$

$$\therefore \quad y(n) = \{3, 3, \underset{\uparrow}{0}, 3, 3\}$$

As shown in Figure (19), each sample is added to the corresponding one.

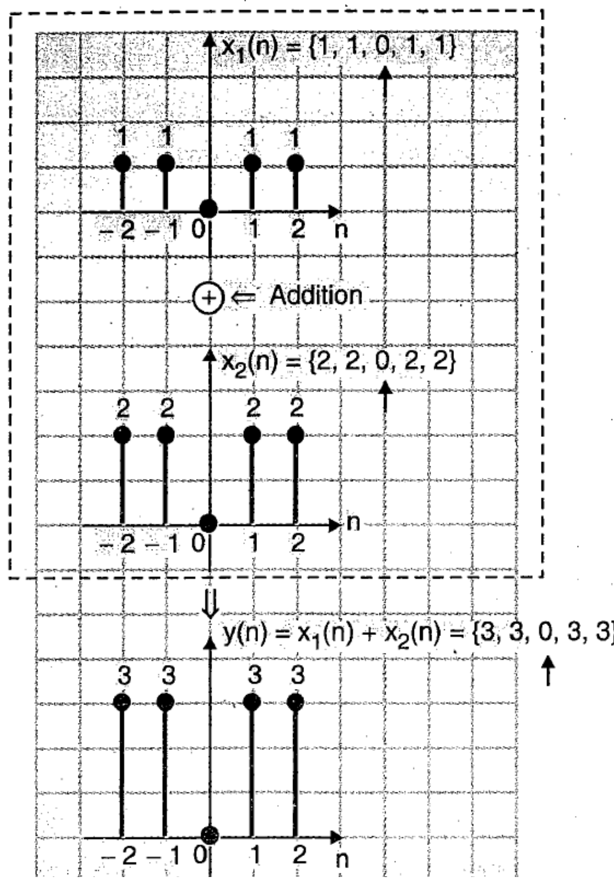


Figure 19: Addition operation

1-3.7 Signals Multiplication

Consider the same sequences $x_1(n)$ and $x_2(n)$, the multiplication of them yields y (n) as shown in Figure (20) and according to the equation:

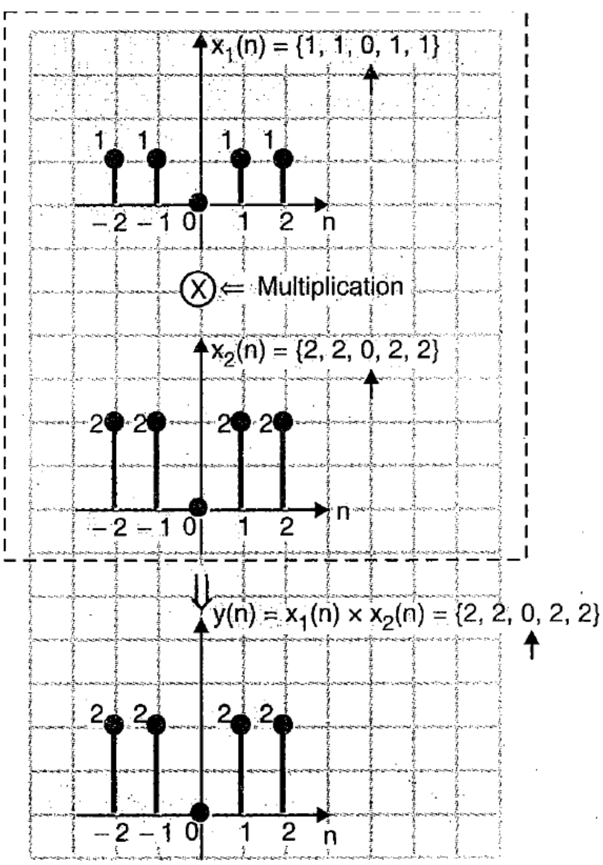


Figure 20: Multiplication operation.

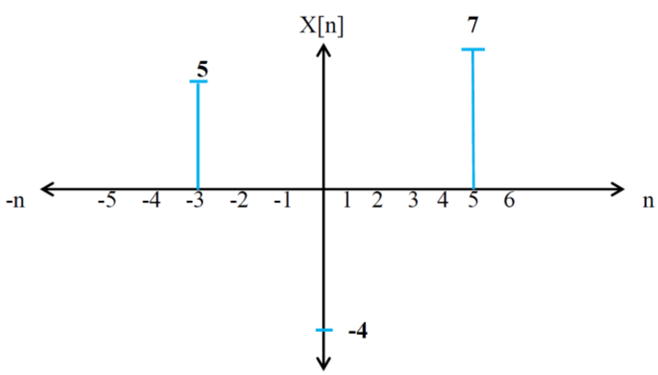
1-4 Examples and Tutorials

Note 1: Properties of Unit Impulse signal are shown in Table (1).

Properties of the Unit Impulse Sequence
1. $x[n]\delta[n] = x[0]\delta[n]$
2. $x[n]\delta[n-k] = x[k]\delta[n-k]$
3. $\delta[n] = u[n]-u[n-1]$
4. $u[n] = \sum_{k=-\infty}^{n} \delta[k]$
5. $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

Example 1: Sketch the signal x[n] =5S[n+3] - 4S[n] + 7S[n-5].

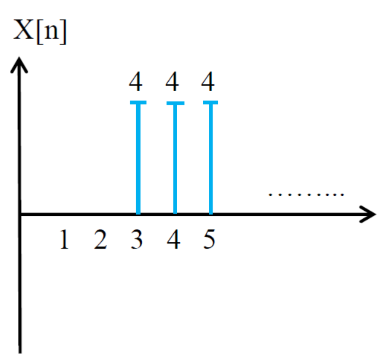
Sol:



Note 2		
x(n+1)	Shift left by 1 <	
x(n-2)	Shift right by 2 >>	
x(-n+3)	Folded then Shift right by 3	
x(-n-4)	Folded then Shift left by 3	
Conclusion	+ + and - -	< left
	+ - and - +	> right

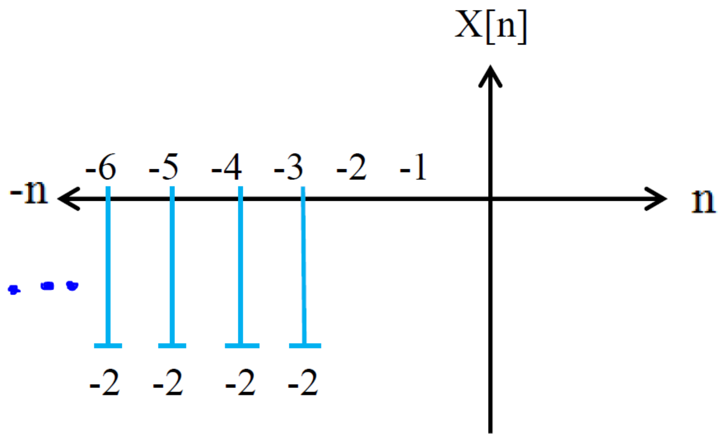
Example 2: Sketch the signal x[n]= 4u[n-3].

Sol:

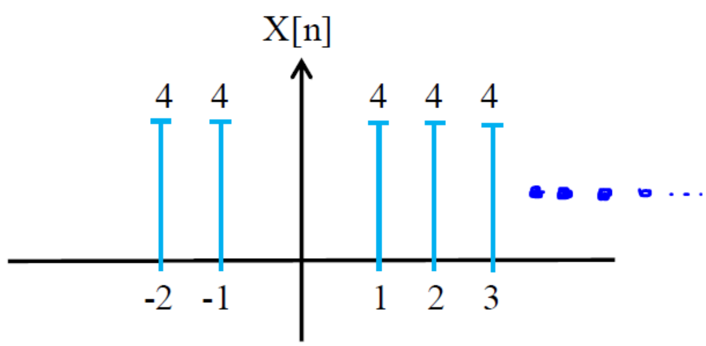


Example 3: Sketch the signal x[n]= -2u[-n-3]

Sol:



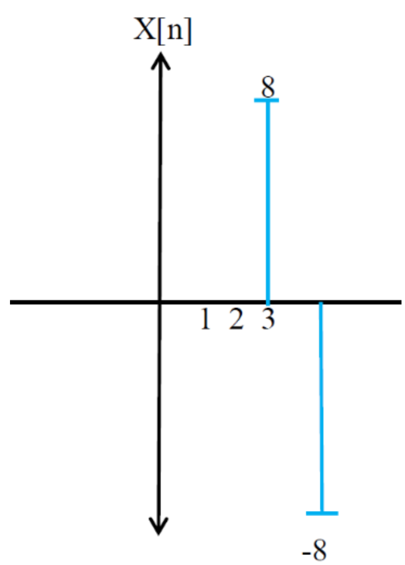
Example 4: Find the expression for signal:



Sol: 4u[n+2]

Example 5: Find the expression for signal:

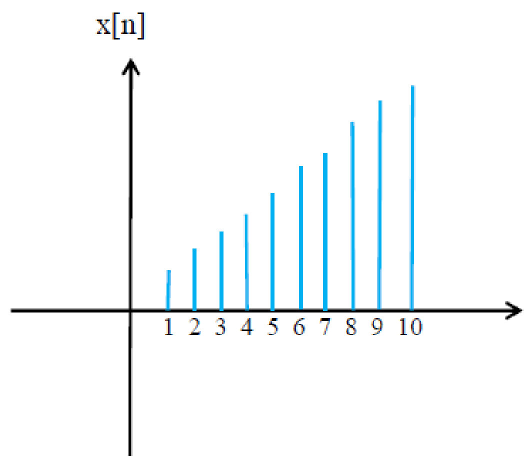
Sol: 8 δ [n-3] - 8 δ [n-4]



Example 6: Sketch the signal $X[n] = e^{0.2n}\, u[n]$:

Sol:

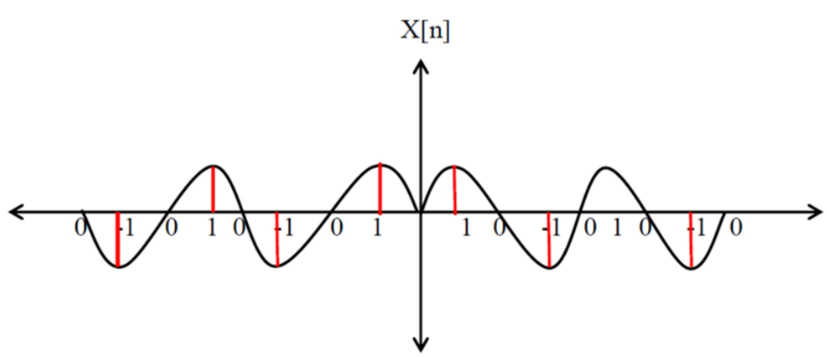
n	x[n]
0	1
1	1.22
2	1.49
3	1.82
4	2.22
5	2.7
6	3.3
7	4.055
8	4.95
9	6.04
10	7.3



Example 7: Sketch the signal $x[n] = \sin\left[\frac{\pi n}{2}\right]$

Sol:

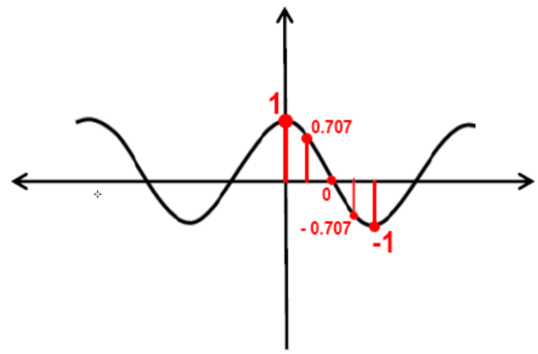
n	x[n]
0	0
1	1
2	0
3	-1
4	0
5	1
6	0
7	-1
8	0
9	1
10	0



Example 8: Sketch the signal $X[n]= \cos [\frac{\pi n}{4}]$

Sol:

n	x[n]
0	1
1	0.707
2	0
3	-0.707
4	-1
5	-0.707
6	0
7	0.707
8	1
9	0.707



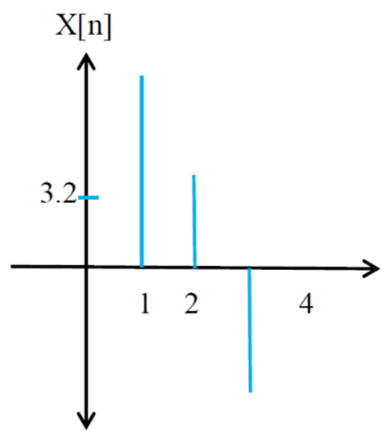
Example 8: Consider the following two sequence of length (5):

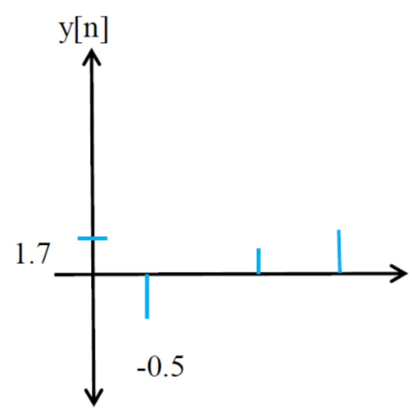
X [n] = { 3.2 , 41 , 36 , -9.5 , 0 }

Y [n] = { 1.7 , -0.5 , 0 , 0.8 , 1 }

Find

a) X [n] . Y[n]
b) X [n] + Y [n]
c) $\frac{7}{2}$ x[n]





Sol: a) Z[n]= x[n].y[n]={5.44,-20.5,0,-7.6,0}

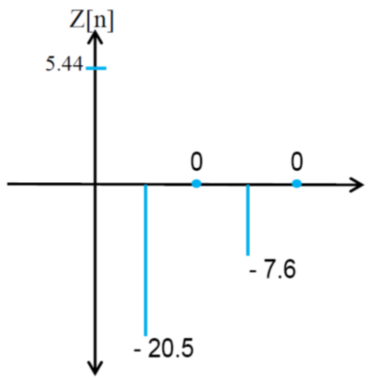
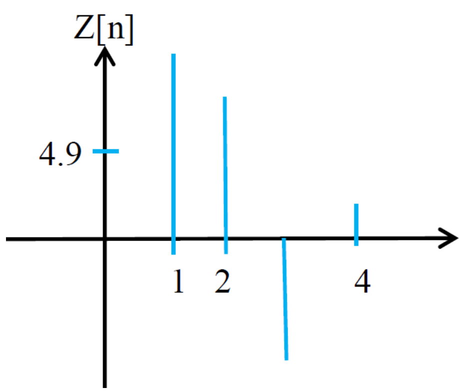
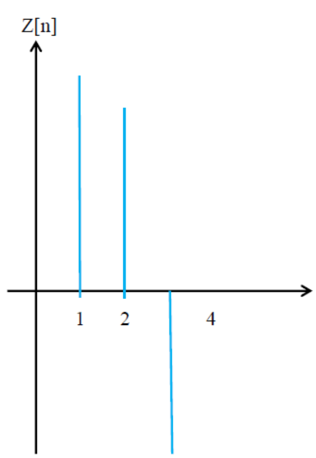


Figure a

b) Z[n]= x[n]+y[n] ={4.9,40.5,36,-8.7,1}



c) Z[n]= {11.2 ,143.5 ,126 ,-33.2 ,0}



Example 9:

Example:- Consider the following two sequence of length (7) defined for $-3\leq n\leq3$

X[n]={3,-2,0,1,4,5,2}

Y[n]={0,7,1,-3,4,9,-2}

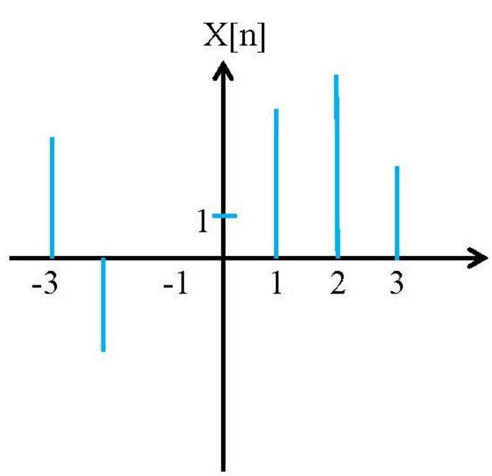
3y[n]={0,21,3,-9,12,27,-6}

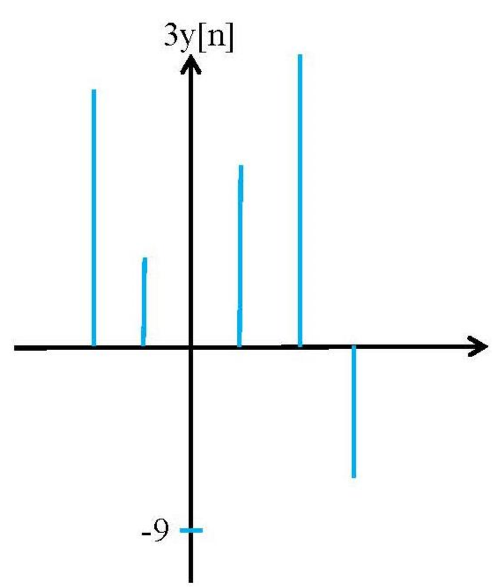
Also g[n]={-5,0,3,1} , $0 \leq n \leq 3$

Find

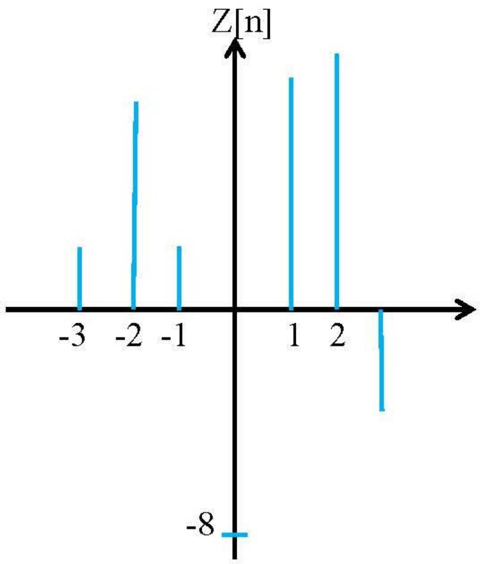
a) z[n]=x[n]+3y[n]
b) z[n]=y[n]-g[n]
c) $z[n]=\frac{x[n]-y[n]}{2}$
d) z[n]=x[n-1]-3y[n+2]

Sol.

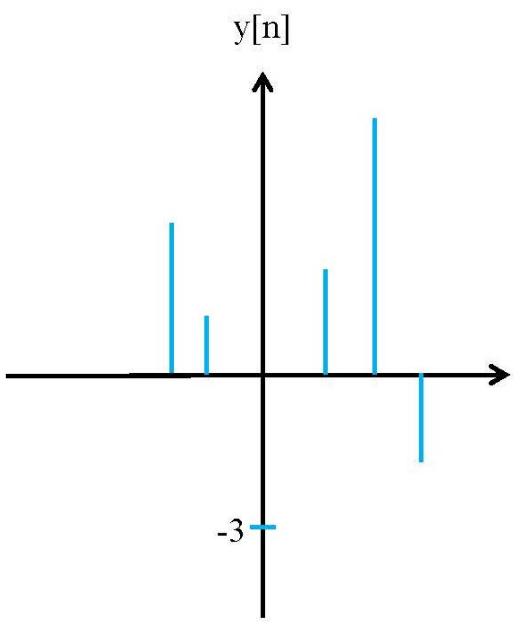


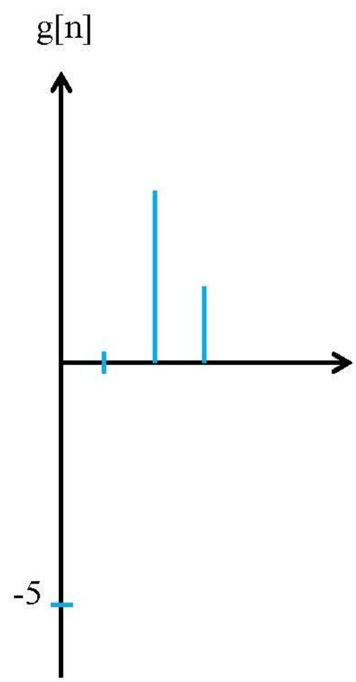


Z[n]=x[n]+3y[n]={3,19,3,-8,16,32,-4}



B)

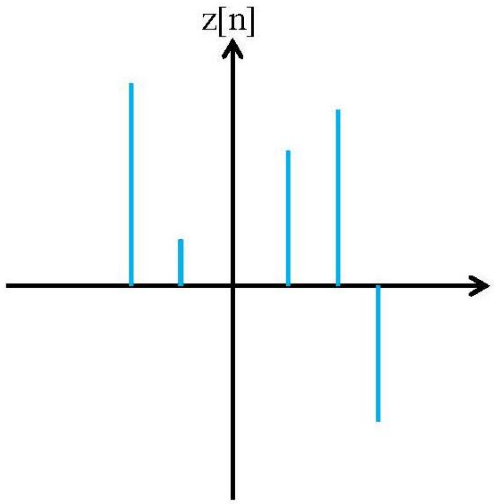




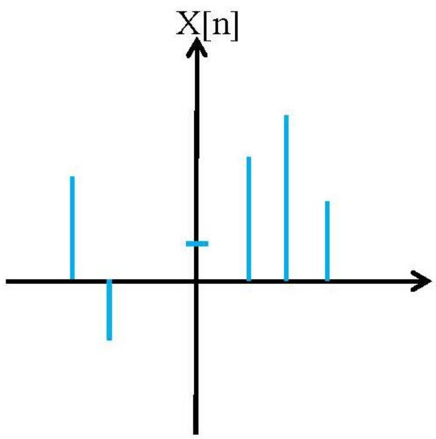
y[n]={ 0,7,1,-3,4,9,-2}

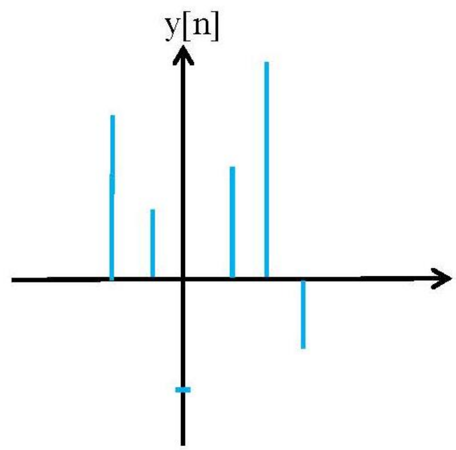
g[n]={0,0,0,-5,0,3,1}

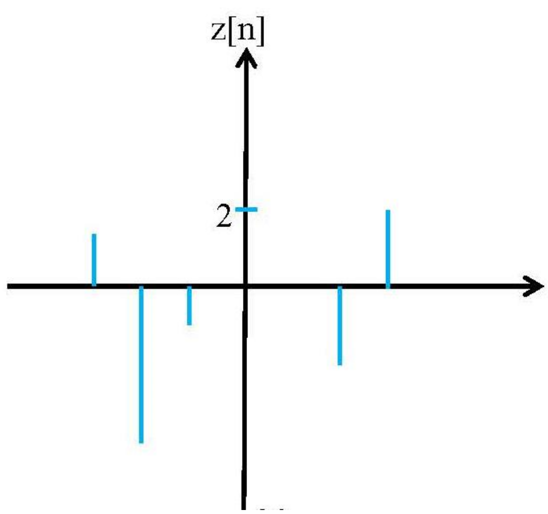
z[n]=y[n]-g[n]={0,7,1,2,4,6,-3}



C)



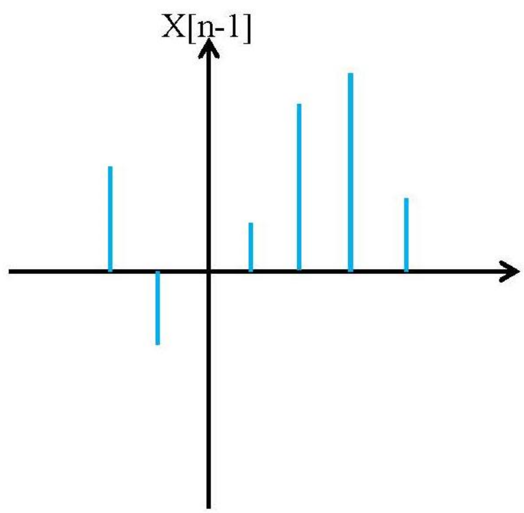


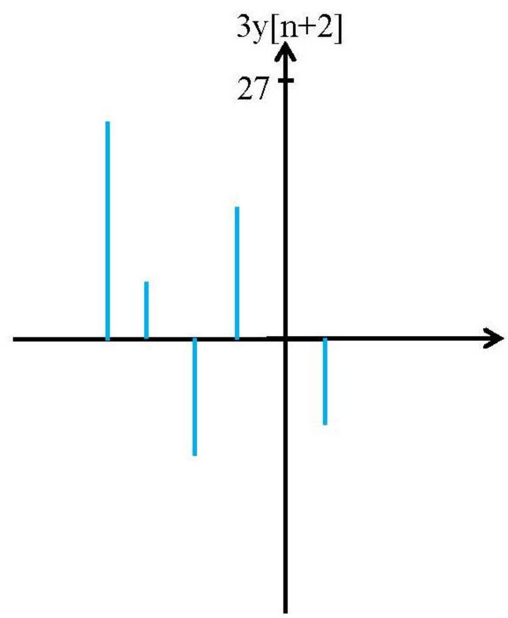


X[n]-y[n]={3,-9,-1,4,0,-4,4}

$\frac{x[n]-y[n]}{2}$ ={1.5,-4.5,-0.5,2,0,-2,2}

D)

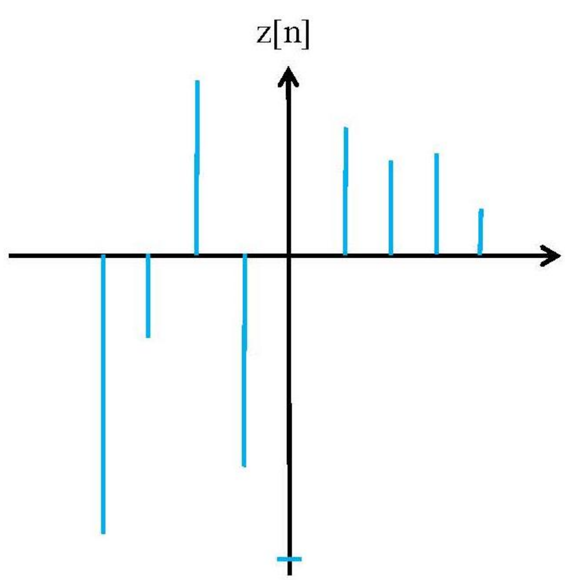




x[n-1]={3,-2,0,1,4,5,2}

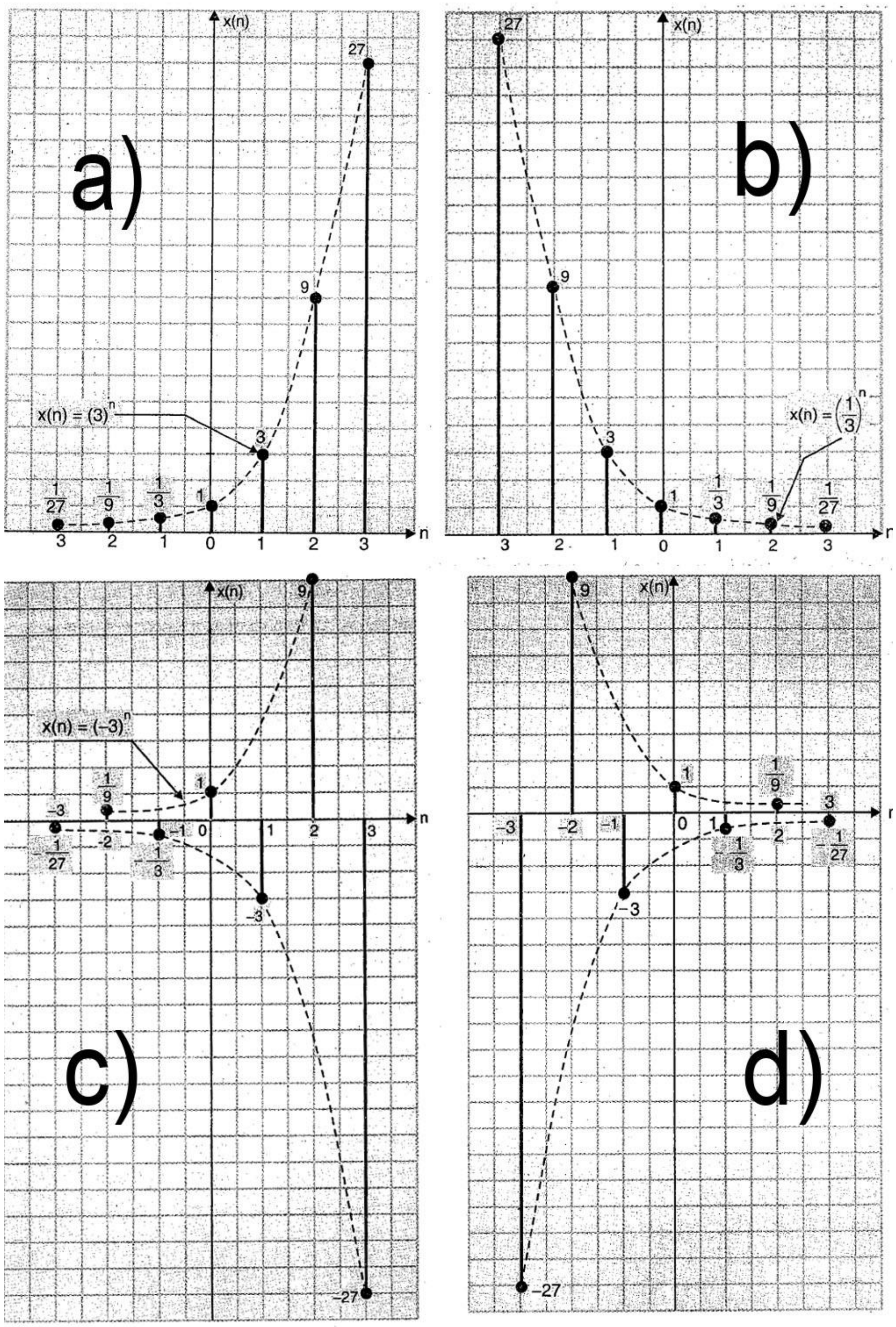
3y[n+2]={0,21,3,-9,12,27,-6}

Z[n]=x[n-1]-3y[n+2]={0,-21,-3,12,-14,-27,7,4,5,2}

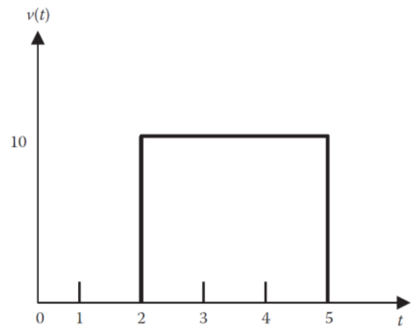


Example 10: Sketch the signal X[n] = a^n for a) a= 3, b) a=1/3, c) a= -3, d) a = -1/3.

Sol:



Example 11: for the figure below, write the output equation.



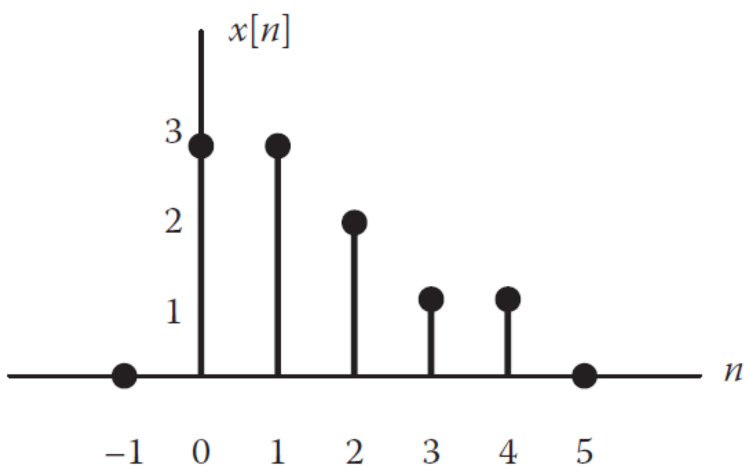
Sol: $v(t) = 10u(t-2) - 10u(t-5)$

Example 12: Sketch each of the following signals:

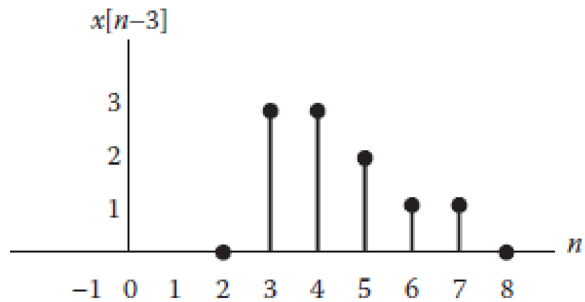
(a) $x[n-3]$, (b) $x[-n+3]$, (c) $x[2n]$.

(d) $x[-n]$, (e) $x[n+2]$, (f) $x[n/2]$

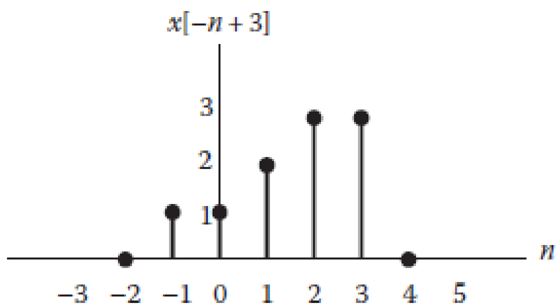
if $x[n]$ is shown in the figure :



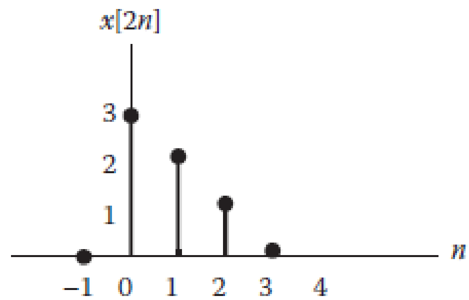
Sol:


x[n–3]
3
2
1
n
–1 0 1 2 3 4 5 6 7 8

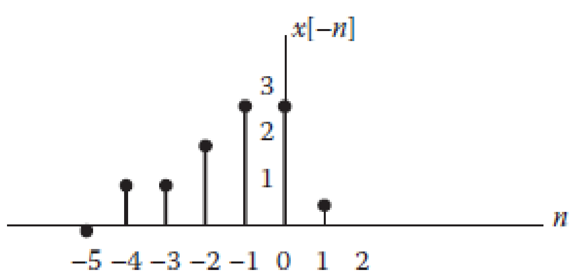
(a)

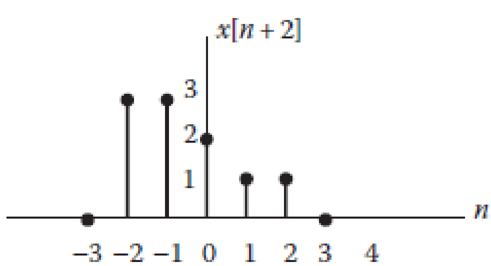

x[–n + 3]
3
2
1
n
–3 –2 –1 0 1 2 3 4 5

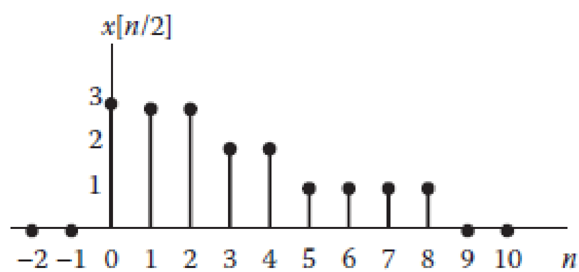
(b)


x[2n]
3
2
1
n
–1 0 1 2 3 4

(c)


x[–n]
3
2
1
n
–5 –4 –3 –2 –1 0 1 2


x[n + 2]
3
2
1
n
–3 –2 –1 0 1 2 3 4


x[n/2]
3
2
1
–2 –1 0 1 2 3 4 5 6 7 8 9 10 n

Example 13: Find expressions for the various signals shown in figure below:

Sol:

(a) x[n] = -2u[-n-4]
(b) x[n] = u[n+3] – u[n-5]
(c) x[n] = 8δ[n-6]
(d) x[n] = 2r[n+6]-2r[n+2]